The importance of early number concepts for learning mathematics in deaf and hard of hearing children

Abstract

This chapter discusses important background knowledge and research findings from a variety of disciplines that inform best practices for supporting optimal mathematical achievement in all children. Studying deaf and hard of hearing children affords an unique lens on numerical cognition and can reveal mechanisms that underlie the relationships between language and numerical cognition in all children. First, discussion will begin with the importance of early numeracy for later academic outcomes, and why prioritization of instruction time and early intervention are needed to increase the likelihood of a strong foundation in numeracy. Second, a brief overview of numeracy development milestones will provide a basis for discussion of our central thesis: language experiences can impact numerical cognition, which then have a significant impact on academic outcomes. Third, given the importance of numeracy skills in academic outcomes, we describe pedagogical trends that are likely to support the development of numerical cognition. Finally, we offer some future directions of research that will further account for underlying mechanisms of numeracy development in very young and preschool-aged children.

Amos G. Draper, the first Deaf mathematics professor at the National Deaf-Mute College (now Gallaudet University), marveled at children's natural curiosity:

> Children, with eyes and ears opened, are filled with admiration by regularity of outline, beauty of color, and harmony of sound…As children grow to [adulthood], the love of form, color, and harmony remains central…Does it not flow from the instinctive but unrecognized perception of mathematical principles? (Draper, 1876)

Draper discussed how a child would learn mathematical principles indirectly and directly and how the knowledge would further propel the child's curiosity about the universe (Kurz, 2006, 2008). In essence, every child is a mathematician if provided with unstructured and structured, irregular and regular activities at home, in school and in the community. Draper's speech, "The Influence of Mathematical Studies upon Personal Character," is now 144 years years old. In this time, what have we learned about mathematics education and young childhood deaf education?

Numeracy Skills and Academic Outcomes

Various measures of academic readiness of children upon school entry and their predictive value of later academic outcomes have received much attention in the educational sciences, with considerable focus on executive function, language, and reading (Bull et al., 2011; M. L. Hall et al., 2019; Henner et al., 2016; Hrastinski & Wilbur, 2016; Mayberry et al., 2011). However, a meta-analysis of six large longitudinal studies assessing a range of cognitive, academic, and social background measures of approximately 52,000 children found that children's mathematical cognition skills at the beginning of schooling was the strongest predictor for their later academic outcomes (both reading and math) in later primary school grades (Duncan et al., 2008). Despite the high impact of mathematical cognition on academic outcomes, the Organisation for Economic Co-operation and Development (OECD) finds approximately one-third of the United States adult population is limited to reading numbers and performing limited, one-step arithmetic operations (OECD levels 1 and below) (OECD, 2013a). The upper third (OECD levels 3-5) have a stronger sense of mathematical relationships (e.g. percentages) and can select optimal problem-solving formulae to interpret options (e.g. comparing health insurance plans). As a result approximately 73 million Americans are not well equipped to make informed decisions about their finances (e.g. cumulative costs of loans) or health (e.g. understanding the probability of infection or likelihood of recovery from medical treatment), which has dire implications for education policy decisions and illustrate the importance of allocating valuable resources towards strong numeracy skills. (OECD, 2013b).

Deaf and hard of hearing (d/hh) children, who have higher incidence rates of reduced language input and fluency, have documented delays relative to typically hearing children in a variety of areas of mathematical reasoning (Kritzer, 2009; Nunes & Moreno, 2002), such as counting (Nunes & Moreno, 1998), word problems (Hyde et al., 2003), fractions (Titus, 1995), arithmetic comparison problems (Kelly & Mousley, 2001). These delays are well documented in Gottardis et al. (2011) which presents a meta-analysis of 23 studies comparing d/hh children and typically hearing children. However, Secada (1984) demonstrates that comparable development of number concepts is observed when comparing deaf children learning ASL and typically hearing children learning spoken English when both groups have similar rote counting skills (i.e., reciting the number words in order without understanding their quantity meaning). This chapter further explores the important role played by a strong language foundation for fostering numeracy skills in d/hh children.

While objective assessments of numerical skills are an important measure, math anxiety and subjective self-evaluation of numerical skills also represent an important dimension in both children (Ganley & Lubienski, 2016) and adults (Peters et al., 2019). Peters et al. (2019) compare adults who are categorized by self-assessed confidence (high and low) and objectively-assessed numerical skills (high and low) on the accuracy of hypothetical medical decisions. Only one of the 4 possible conditions resulted in optimal health decisions. That is, high numeracy abilities alone are not sufficient; alignment of both high confidence and high numeracy abilities were necessary for optimal health decisions (Peters et al., 2019). Thus, best pedagogical practices should include an objective to ensure the student's confidence and skills are aligned (Fives et al., 2014).

Numeracy Developmental Milestones

A large literature now supports the existence of two subsystems for representing quantities that do not depend on language, and that are phylogenetically shared between human and non-human animals. However, each of these systems has limitations, as described below. Humans, even as infants, utilize the object tracking system (OTS) to precisely distinguish between small quantities up to four, (e.g. two versus three items), a process known as *subitizing*. In addition to subitization, infants can also approximate distinctions between larger quantities (e.g., six versus twelve) with the approximate number system (ANS) (Halberda & Feigenson, 2008; Xu & Spelke, 2000). Infants are not alone in this ability to demonstrate numerical cognition without having acquired understanding of number words; a wide range of animal species have also shown sensitivity to object numerosity. For example, fish, birds, non-human primates, and equines also exhibit OTS and ANS abilities (Agrillo et al., 2014; Cantlon et al., 2016; Emmerton et al., 1997; Gabor & Gerken, 2014; Pepperberg, 1994).

These numerical capabilities observed across animal species and young infants are interpreted as support for the existence of phylogenetic systems that represent and process numerical quantities independently of language (Halberda & Feigenson, 2008). However, another system is required to exactly represent quantities outside of the subitizable range (that is, quantities larger than 4). Most (but not all) human languages have a count list, that is, a sequence of words or signs that refer to the natural numbers (Butterworth et al., 2011; Corbett, 2000). In industrialized, numerate societies, children typically learn number words first as merely counting routines to be recited in sequence (similar to rehearsed games like "patty-cake".) Thus, young children (typically younger than age 4) may appear to be able to count to relatively high numbers, such as twenty, but there is a dissociation between ability to recite number words in sequence, and comprehension of number word meaning (Fuson, 1991; Sarama & Clements, 2019).

While there is considerable variability in the timing, around the age of 2 years, children learn that the linguistic symbol "one" refers to a single object (Carey, 2009). Upon achieving this developmental milestone, the child is then referred to as a "one-knower." In the classic "Give-a-Number" experimental paradigm, children will be able to correctly give one object when asked for one object, and provide an incorrect number of objects when asked for two or any other larger quantity (Wynn, 1990). Children remain at this stage for multiple months before progressing to the two-knower level. The three-knower and four-knower levels subsequently follow, also with significant time elapsing between these stages. When the four-knower level is achieved children are generally able to successfully implement the cardinality principle. The cardinality principle refers to children's understanding that the last number word used in tagging a set of objects reflects a property of the set, and does not just apply to that object (e.g., (Fuson, 1988).. Once they learn the cardinality principle (aka, become “CP-knowers”) they implicitly understand that when counting, (A) each word represents a specific quantity, (B) each object is labeled with a number word only once, and (C) each number word must be said in the correct order. As CP-knowers, children are able to apply the cardinality principle to additional number words and rapidly expand their ability to represent larger quantities with precision (Carey, 2009).

Number Development and Language Experience

While the developmental sequences outlined above appear to be universal in order, the timing of these milestones varies across and within linguistic and cultural groups and effects are observed to persist across the developmental spectrum up to adulthood. Differences in early life experiences with pedagogical approaches (Pagliaro, 2010), specificity of the lexicon for number (Gordon, 2004; Pica et al., 2004), and parental expectations and use of number language (Elliott & Bachman, 2018) have all been identified as sources of these individual and group-level variations in numerical cognition. Thus, it is clear that mathematical cognition is not unitary and multiple domains, language in particular, contribute to numeracy performance (Carey, 2009; Levine & Baillargeon, 2016). Here, the focus is on the connections between language experience and numerical cognition. The multiple ways language affects numerical cognition across a range of populations and their specific language experiences reveals a robust mechanism that supports numeracy skills in all children, regardless of their auditory status or the modality(ies) of their language(s).

Globally, comparisons between deaf, hard-of-hearing, and typically hearing students' numerical cognition have shown d/hh children lagging behind typically hearing peers (Gottardis et al., 2011; Traxler, 2000). For example, Kelly and Mousley (2001) find that d/hh and typically hearing college students' arithmetic skills are similar, however, typically hearing college students tended to perform better than d/hh college students when confronted with word problems. Kelly and Mousley utilized an experimental design that allowed them to dissociate several skills: reading, basic arithmetic, comprehension of numerical relationships, and motivation to solve problems. By dissociating these factors, they interpret the performance differences between d/hh and typically hearing children as arising not from literacy differences, but instead from pedagogical differences and expectations. Pedagogical practices of teachers of d/hh students appear to focus on rote practice of arithmetic rather than problem solving, which requires sophisticated use of mathematical language (Easterbrooks & Stephenson, 2006; Kelly et al., 2003; Ottem, 1980). These delays have a cascading effect throughout development and throughout life: The proportion of DHH people employed in Science, Technology, Engineering, and Mathematics (STEM) disciplines is very small (0.13–0.19%) compared to that of the general population (11–15.3%) (National Center for Science and Engineering Statistics (US), 2011). While the importance of accessible linguistic input, whether signed or spoken, to language development and academic success has long been acknowledged, attention to the role of language in mathematics achievement has been underappreciated and under-researched. The number of studies exploring number concept development and mathematical achievement in d/hh children is small; further, such studies rarely report or consistently control children's language experiences. While Pagliaro and Kritzer (2013) have suggested that exposure to signing deaf parents or adults increases d/hh children's “incidental learning opportunities” at home and in school, the specific role played by language in the development of number concepts in d/hh children has not been examined systematically (see Gottardis, et al., 2011 for a discussion and Carrigan et al., in prep for a study design that dissociates hearing status and language experience). Recent work has increasingly focused on the importance of early language development in children's later academic success (Borgna et al., 2018; Dietz, 1995; Risley & Hart, 2006; Snow, 2002; Vukovic & Lesaux, 2013; Weisleder & Fernald, 2013). We advocate bringing this same approach to the study of number concept development and mathematics achievement in d/hh children, beginning as early in development as possible (Cohrssen & Page, 2016).

Number Development and Language-specific Impacts

While all observed children go through these knower-levels in the same sequence, the timing of this developmental trajectory does slightly vary as a result of experiences with specific types of languages (Almoammer et al., 2013; Barner et al., 2009; Piantadosi et al., 2014; Sarnecka, 2014; Sarnecka et al., 2007). For example, grammatical number encodes numerosity and has been shown to affect the timing and trajectories of children's numerical development. Grammatical number in English is expressed by adding the plural morpheme "-s" to a noun (e.g., "cats"), making it a singular/plural language. Other classifications include non-singular/plural (e.g., Japanese and Mandarin Chinese) and singular/dual/plural (Slovenian and Saudi Arabic). While all children learn the meaning of "one", children learning languages with only singular/plural marking remain one-knowers longer than children speaking singular/dual/plural languages, who move more quickly to (and stay longer at) the two-knower level. Children learning singular/dual/plural languages are argued to receive the benefits of more extensive grammatical number marking (Sarnecka, 2014).

Language that parents use with children involving counting or labeling sets of visible objects is related to children's later ability to connect the appropriate quantity with the number word, and to their understanding of the cardinality principle (Levine et al., 2010). Parental talk about sets involving 4 to 10 objects more strongly predicted children's later cardinal-number knowledge than did talk about smaller sets (Gunderson & Levine, 2011). Number language produced by preschool teachers is also related to the amount of growth in children's number knowledge over the school year (Klibanoff et al., 2006). Number words are not the only type of language that have a positive impact on the development of number concepts. Children in a Head Start program who received a dialogic book-reading intervention focused on mathematical language improved in their number knowledge more than a control group who received regular instruction. Examples of mathematical language included words and phrases like "a lot," "more," "inside," and "near" (Purpura et al., 2017). Notably, both signed and spoken languages employ linguistic devices (e.g., reduplication) to encode aspects of number, such as plurality and magnitude (Corbett, 2000; Kurz & Pagliaro, 2019).

Numeracy Development in Impoverished Language Environments

The nature of the relationship between numeracy and other domains of cognition remains under debate (Hohol et al., 2017). Numerical cognition research has often pointed to domain-specific systems as the basis for mathematical skills (Feigenson et al., 2004). As described above, the approximate number system (ANS) is posited to be a lower-order, nonverbal process that scaffolds and accounts for higher-order numeracy outcomes. However, it is becoming increasingly apparent that language may be influential in the development of this system (Santos et al., 2019, 2020). When asked to identify "which of two sides of a computer screen contained 'more' dots", d/hh children between the ages of 3 and 6 years who were acquiring only spoken English (i.e., no experience with a signed language) showed lower accuracy than did typically hearing children. However, they performed comparably to the typically hearing children on a version of the same task, using the same dot displays, that did not use any linguistic instructions. Further, when the ages of the d/hh children were adjusted to account for their later access to spoken English, their performance was similar to that of (younger) typically hearing children (Santos et al., 2019).

Indeed, Santos et al. (2020) is the first systematic exploration of how language experience--namely the timing of language input (beginning at birth vs. beginning later in development) and language modality (spoken English vs. ASL)--may influence the development of ANS acuity in d/hh children. They found that children who were exposed to spoken or signed language later in development showed poorer ANS acuity than did children who were d/hh and typically hearing who began learning their first language at birth. However, their task instructions were linguistic, leaving open the possibility that language experience affected children's understanding of the task, and not their actual ANS acuity. A subsequent analysis indicated that most of the children showed patterns of performance indicating that they did understand the task (i.e., they performed better on trials with larger differences between the two set sizes (e.g., 9 vs. 3 items, a 3:1 ratio) than they did on trials in which the two sets were closer in quantity (e.g., 13 vs. 10 items, or a 1.3:1 ratio). Overall, such findings are suggestive that language is important for the development of non-symbolic discrimination between large sets, which has historically not been considered to rely on language. It is possible that a certain amount or type of language input or experience is required to support ANS development; the reason that the influence of language has not been detected in previous work might be that children who are typically hearing are practically guaranteed to exceed that threshold, while some d/hh children may not.

Acquiring a counting sequence from a language model early in development appears to be crucial for developing certain types of number representations. Deaf adults in Nicaragua who have not attended school or become part of the Deaf community, and therefore did not learn the count sequence of Nicaraguan Sign Language (NSL), struggle to exactly represent quantities above 5. These adults, called homesigners,[1] are unable to reliably generate or match sets containing 5 or more items (Spaepen et al., 2011). Flaherty & Senghas (2011) found that NSL signers who began learning NSL after early childhood also struggle to exactly represent quantities above 6. These observations are not limited to deaf people or those in emerging sign language communities. Typically hearing adults whose native language does not have specific words that refer to exact quantities in this way (such as "one", "two", "three", etc. in English) also have difficulty matching and representing quantities larger than 4 (Gordon, 2004; Pica et al., 2004).

Thus, we propose that the relative weakness of the mathematical abilities of d/hh children suffer from a serious confound, and that this observed weakness is due to the studies' failure to account for the documented variations in d/hh children's language experiences and the implications for their number representations. Here, we propose the best interpretation of numeracy development is optimally made with consideration of how some kinds of numeracy depend on language. By examining the effects of language deprivation (W. C. Hall, 2017) on numeracy development, we can better understand numeracy outcomes across a broad spectrum of populations (e.g., bilinguals with later formal L2 instruction in mathematics, children with significantly impoverished L1 input, like d/hh children or children in malnourished

[1] See Coppola (2020) for a brief profile of adult homesigners in Nicaragua and Nicaraguan Sign Language. It is important to note that, as in many other low-income countries around the world, only about 5% of deaf people in Nicaragua attend school, are exposed to language, and participate in the Deaf community.

environments). With this improved understanding, we can better implement targeted interventions and curriculum policy decisions.

Language and Pedagogical Approaches

While national and state level standards establish targets for P-12 mathematics learning, day-to-day implementation of pedagogical strategies are made by teachers and there are a multitude of pedagogical philosophies and approaches in use (Easterbrooks & Stephenson, 2006). These different practices have received considerable attention, though they remain woefully understudied, with nearly all identified approaches classified as requiring additional empirical research to properly test their claims (Beal-Alvarez & Cannon, 2014; Easterbrooks & Stephenson, 2006). Easterbrooks & Stephenson (2006) identified ten different practices relevant for science and mathematics instruction which can be categorized into four themes, (A) language, (B) instructor expertise, (C) critical thinking, and (D) technology and supporting materials. The Easterbrooks & Stephenson (2006) article is complemented by a second publication presenting a survey of master instructors and their evaluations of the importance of the same ten practices (Easterbrooks et al., 2006). Nearly all master instructors surveyed agree that high skills in communicating science and math are of paramount importance. In addition to the perception of the importance of language by master instructors, discussion of pedagogical practices relating to language have received tremendous attention. Careful assessment of efficacy, however, is sorely lacking.

Given that development of core numerical cognition and learning of higher-order arithmetic processes are influenced by language, findings that better mathematical performance was observed in children with robust access to language is unsurprising. D/hh students who were unable to understand spoken presentation of instructional content were unable to solve the prompted math questions (Enderle et al., 2020; Serrano Pau, 1995). The role of language fluency of the instructor is further elevated considering that d/hh children have fewer opportunities for incidental learning because relatively few of their parents are fluent in a signed language (Nunes & Moreno, 1998). The teacher must then be capable of producing a fluent utterance that conveys meaningful mathematics concepts (Schindler & Davison, 1985).

Number and Math Interventions

Researchers and educators have developed multiple practices for preschool and elementary school children to improve their number knowledge and mathematical performance (without regard to deafness and its contextual factors) (c.f. Frye et al., 2013). Further studies assessing the efficacy of numeracy interventions, especially with very young d/hh children who have not yet begun formal schooling, are very much in need. Following up on work showing that d/hh children begin kindergarten without the requisite numeracy foundation (e.g., Kritzer & Pagliaro, 2012; Pagliaro & Kritzer, 2013) carried out an intervention, building the Math Readiness Parents as Partners (MRPP) project, that was designed to increase parental behaviors that are known to support the development of foundational mathematics concepts. Their efforts to train parents to mediate early mathematics concepts with their d/hh children were successful in increasing parents' use of mathematics and related vocabulary and in other measures of parent-child interactions. However, the causal impact of specific parental behaviors on the children's mathematics performance was not evaluated.

One ongoing project examines the impact of dramatically increasing the amount of number words and counting behaviors experienced by d/hh children between the ages of 2.5 and 5.5

years (Coppola, 2016). In this design, parents will read specially designed number books with their children, and encourage the children to count sets of objects ranging from 1 to 10. These children will be compared to similar children who have been randomly assigned to read books that focus on the associations between letter names (e.g., “bee”) and the shapes of letters (e.g., B). Outcome measures include the difference between pre- and post-training performance on the Give-a-Number task, as well as the Which-is-X task, in which children see two sets of the same type of object (e.g., 4 birds vs. 5 birds) and are asked to “point to 4”. If exposure to number words and counting itself drives the development of number knowledge, including cardinality, Coppola and colleagues expect the children in the number-book reading condition to show greater growth relative to children in the letter-book group (controlling for other factors that are known to influence number knowledge growth, such as age, general vocabulary, executive functioning, and socioeconomic status).Their predictions are supported by recent work from their laboratory (Carrigan et al., in prep.; Walker et al., 2021) The researchers plan to develop techniques to widen the scope of participation to a national level, for example, developing a website to encourage parents across the US to use more number language with their children, and innovating a tablet-based game application that would assess the impact of parental number language on children’s number knowledge. These interventions are focused on the predictive power of early numeracy on later academic outcomes.

Summary

Here, we have provided a succinct overview of the major elements of numeracy development, from number processing without language of small precise quantities and larger approximate quantities to language-dependent representations of large precise quantities. This process is not straightforward and rapid, in fact, it typically requires children 2 years to be able to decipher the rules underlying the cardinality principle and then apply the cardinality principle to numbers greater than 4. Strong numeracy skills consistently predict positive academic outcomes, pointing to the need for prioritizing numeracy instruction time to help ensure all children have a strong foundation in numeracy skills.

As shown from our review of the literature, the past twenty years highlight the impact of language input and children’s linguistic fluency on their numeracy foundation in preschool and their later academic outcomes. Some authors have articulated a rights-based argument for putting into place early childhood education practices that provide all children with a solid foundation in numeracy (see Cohrssen & Page, 2016). Based on this literature, we describe policy and curriculum practices that optimally support the development of numerical cognition. These practices can be further informed by ongoing research that will elucidate the underlying mechanisms of numeracy development in very young and preschool-aged children, as well as specific pedagogical practices that can foster such learning in older school-aged children. As Draper stated 144 years ago, which is still relevant now, every child today has a natural and innate curiosity about the world and they need to be provided with natural, structured and unstructured, regular and irregular activities at home, in school and elsewhere, to build mathematical knowledge. Natural language learning practices reinforce their early mathematical concepts, including number sense.

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